Appendix A. Derivation of equation (13)

Here we derive the integral notation of (11). We first note that $\overline{s}\left(n_i,a_i\right)=0$ for $a_i>n_i$. Therefore, (11) can be written as

$$\begin{aligned} P\left[\overrightarrow{D}|\theta,m,J\right] &= \frac{J!}{\prod_{i=1}^{S} n_i \prod_{j=1}^{J}\Phi_j!}\frac{\theta^S}{(I)_J}\sum_{A=S}^{J}K(\overrightarrow{D},A)\frac{I^A}{(\theta)_A}= \\ &= \frac{J!}{\prod_{i=1}^{S} n_i \prod_{j=1}^{J}\Phi_j!}\frac{\theta^S}{(I)_J}\left(\prod_{i=1}^{S}\sum_{a_i=1}^{n_i}\overline{s}\left(n_i,a_i\right)\frac{\overline{s}\left(a_i,1\right)}{\overline{s}\left(n_i,1\right)}\right)\frac{I^A}{(\theta)_A}= \\ &= \frac{J!}{\prod_{i=1}^{S} n_i \prod_{j=1}^{J}\Phi_j!}\frac{\theta^S}{(I)_J}\sum_{a_1=1}^{n_1}\dots\sum_{a_S=1}^{n_S}P_1\frac{I^{A_1}}{(\theta)_{A_1}} \qquad \text{(A-1)} \end{aligned}$$

where

$$A_i := \sum_{j=i}^{S} a_i \qquad \text{(A-2)}$$

(so $A_1=A$) and

$$P_i := \prod_{j=i}^{S}\overline{s}\left(n_j,a_j\right)\frac{\overline{s}\left(a_i,1\right)}{\overline{s}\left(n_i,1\right)} = \prod_{j=i}^{S}\overline{s}\left(n_j,a_j\right)\frac{\Gamma\left(a_j\right)}{\Gamma\left(n_j\right)} \qquad \text{(A-3)}$$

We write the first summation of (A-1) in terms of P_2, A_2 and a_1:

$$\sum_{a_1=1}^{n_1}P_1\frac{I^{A_1}}{(\theta)_{A_1}} = \sum_{a_1=1}^{n_1}P_1\frac{I^{A_2+a_1}}{(\theta)_{A_2+a_1}} = P_2\sum_{a_1=1}^{n_1}\overline{s}\left(n_1,a_1\right)\frac{\Gamma\left(a_1\right)}{\Gamma\left(n_1\right)}\frac{I^{A_2+a_1}}{(\theta)_{A_2+a_1}} \qquad \text{(A-4)}$$

Expressing $(\theta)_{A_2+a_1}$ in terms of Gamma functions and noting that $\Gamma\left(n_1\right)=\left(n_1-1\right)!$, we obtain after rearranging terms

$$\sum_{a_1=1}^{n_1}P_1\frac{I^{A_1}}{(\theta)_{A_1}} = P_2 I^{A_2}\frac{1}{\left(n_1-1\right)!}\frac{\Gamma\left(\theta\right)}{\Gamma\left(\theta+A_2\right)}\sum_{a_1=1}^{n_1}\overline{s}\left(n_1,a_1\right)I^{a_1}\frac{\Gamma\left(a_1\right)\Gamma\left(\theta+A_2\right)}{\Gamma\left(\theta+A_2+a_1\right)} \qquad \text{(A-5)}$$

The last quotient is the Beta function that can be written in its integral form:

$$\sum_{a_1=1}^{n_1}P_1\frac{I^{A_1}}{(\theta)_{A_{i1}}} = P_2 I^{A_2}\frac{1}{\left(n_1-1\right)!}\frac{\Gamma\left(\theta\right)}{\Gamma\left(\theta+A_2\right)}\sum_{a_1=1}^{n_1}\overline{s}\left(n_1,a_1\right)I^{a_1}\int_0^1 x_1^{a_1}\left(1-x_1\right)^{A_2}\frac{\left(1-x_1\right)^{\theta-1}}{x_1}dx_1 \qquad \text{(A-6)}$$

Changing the order of summation and integration and using Pochhammer notation leads to:

$$\begin{aligned} \sum_{a_1=1}^{n_1}P_1\frac{I^{A_1}}{(\theta)_{A_1}} &= \int_0^1\frac{1}{\left(n_1-1\right)!}P_2\frac{I^{A_2}}{(\theta)_{A_2}}\left(1-x_1\right)^{A_2}\frac{\left(1-x_1\right)^{\theta-1}}{x_1}\sum_{a_1=1}^{n_1}\overline{s}\left(n_1,a_1\right)\left(Ix_1\right)^{a_1}dx_1 = \\ &= \frac{1}{\left(n_1-1\right)!}\int_0^1 P_2\frac{\left(I\left(1-x_1\right)\right)^{A_2}}{(\theta)_{A_2}}\frac{\left(1-x_1\right)^{\theta-1}}{x_1}\left(Ix_1\right)_{n_1}dx_1 \qquad \text{(A-7)} \end{aligned}$$

This means that the summation of $P_1\frac{I^{A_1}}{(\theta)_{A_1}}$ over a_1 is written in terms of an integral over $P_2\frac{\left(I\left(1-x_1\right)\right)^{A_2}}{(\theta)_{A_2}}$ and some additional terms that do not depend on any a_i. More generally,

$$\sum_{a_i=1}^{n_i}P_i\frac{\left(I\prod_{k=1}^{i-1}\left(1-x_k\right)\right)^{A_i}}{(\theta)_{A_i}} = \frac{1}{\left(n_i-1\right)!}\int_0^1 P_{i+1}\frac{\left(I\prod_{k=1}^{i}\left(1-x_k\right)\right)^{A_{i+1}}}{(\theta)_{A_{i+1}}}\left(Ix_i\prod_{k=1}^{i-1}\left(1-x_k\right)\right)_{n_i}\frac{\left(1-x_i\right)^{\theta-1}}{x_i}dx_i \qquad \text{(A-8)}$$

for $i = 1...S$. The S summations in (A-1) require repeated application of (A-8), from $i = 1$ to $i = S$, which yields

$$P\left[\overrightarrow{D}|\theta,m,J\right] = \frac{J!}{\prod_{i=1}^{S} n_i! \prod_{j=1}^{J} \Phi_j!} \frac{\theta^S}{(I)_J} \int_0^1 ... \int_0^1 \prod_{i=1}^{S} \left(Ix_i \prod_{k=1}^{i-1} (1-x_k) \right)_{n_i} \frac{(1-x_i)^{\theta-1}}{x_i} dx_1 ... dx_S \quad \text{(A-9)}$$

which is (13) with (14).

Appendix B. The relation of the approximation (10) to the exact result (6)

A rigorous expansion of the approximate result (10) in Vallade & Houchmandzadeh (2003) in terms of $(1/J_{\mathrm{M}})$ powers can be written after some algebra (Alonso & McKane, *unpublished*):

$$S\left(n\right) = S_0(n) + \mathcal{O}\left(\frac{1}{J_M}\right) \tag{B-1}$$

where $S(n)$ is given by (10) and $S_0(n)$ is given by (6). This expansion confirms that Vallade & Houchmandzadeh's result (10) converges to (6) when J_{M} tends to infinity. The existence of this expansion suggested a quantitative path to independently estimate metacommunity sizes and biodiversity numbers (and hence also speciation rates) from species-abundance data. In the main text we have shown that expected sampling abundances do not depend on metacommunity size. Therefore, that initial hope of independent estimation must be abandoned. In addition, Vallade & Houchmandzadeh's approximation is not very useful anyway, because the integral in 6 can be evaluated much faster and more robustly than the sum in (10).

The complete proof for the expansion given in (B-1) will be given elsewhere, but an easy argument to show to what extent (10) and (6) converge to each other as J_{M} increases goes as follows. The sum in (10) corresponds to an approximate discretization of the exact integral result given by (6). Let us write (6) as

$$S_0(n) = \int_0^1 G(x)\, dx \tag{B-2}$$

where $G(x) = P_{\mathrm{bin}}^{\mathrm{DL}}[n|m, x, J]\, \Omega(x)$. We divide the interval $(0,\ 1)$ in J_{M} points to obtain

$$S_0(n) \approx \sum_{j=1}^{J_{\mathrm{M}}} G(x_j) \Delta x \tag{B-3}$$

where $\Delta x = 1/J_{\mathrm{M}}$, $x_j = j\Delta x$ and

$$G(x_j) = P_{\mathrm{bin}}^{\mathrm{DL}}[n|m, \frac{j}{J_{\mathrm{M}}}, J]\Omega(\frac{j}{J_{\mathrm{M}}}) \tag{B-4}$$

Hence $S_0(n)$ can be approximated by the following sum:

$$S_0(n) \approx \sum_{j=1}^{J_{\mathrm{M}}} P_{\mathrm{bin}}^{\mathrm{DL}}[n|m, \frac{j}{J_{\mathrm{M}}}, J] \frac{\theta}{j} \left(1 - \frac{j}{J_{\mathrm{M}}}\right)^{\theta - 1} \tag{B-5}$$

Compare this to Vallade & Houchmandzadeh's (2003) formula, given by (10) but repeated here for easier comparison:

$$S(n) = \sum_{j=1}^{J_M} P_{\mathrm{bin}}^{\mathrm{DL}}[n|m, \frac{j}{J_{\mathrm{M}}}, J] E[S_j|\theta, J_{\mathrm{M}}] \tag{B-6}$$

Because each term $E[S_j|\theta, J_{\mathrm{M}}]$ can be approximated by (Alonso & McKane 2004)

$$E[S_j|\theta, J_{\mathrm{M}}] = \frac{\theta}{j}\left(1 - \frac{j}{J_{\mathrm{M}}}\right)^{\theta-1} + \mathcal{O}\left(\frac{1}{J_{\mathrm{M}}^2}\right), \tag{B-7}$$

we conclude that (10), apart from vanishingly small terms, corresponds to the discretization (B-5) in J_{M} points, which by definition converges to the integral in the limit of an infinite number of points.

Appendix C. Proof of the equality of (23) and (29)

Here we show that equations (23) and (29) are identical:

$$\begin{aligned}
E\left[S_n|\theta,m,J\right] &= \frac{\theta}{\left(I\right)_J}\binom{J}{n}\int\limits_0^1\left(Ix\right)_n\left(I\left(1-x\right)\right)_{J-n}\frac{\left(1-x\right)^{\theta-1}}{x}dx = \\
&= \frac{\theta}{\left(I\right)_J}\binom{J}{n}\sum_{j=1}^{n}\sum_{k=1}^{J-n}\overline{s}\left(n,j\right)\overline{s}\left(J-n,k\right)I^{j+k}\frac{\Gamma\left(j\right)\Gamma\left(k+\theta\right)}{\Gamma\left(j+k+\theta\right)} = \\
&= \frac{\theta}{\left(I\right)_J}\binom{J}{n}\sum_{A=1}^{J}\sum_{a=1}^{n}\overline{s}\left(n,a\right)\overline{s}\left(J-n,A-a\right)I^{A}\frac{\Gamma\left(a\right)\Gamma\left(A-a+\theta\right)}{\Gamma\left(A+\theta\right)} = \\
&= \binom{J}{n}\sum_{A=1}^{J}\sum_{a=1}^{n}\overline{s}\left(n,a\right)\overline{s}\left(J-n,A-a\right)\frac{I^{A}}{\left(I\right)_J}\frac{\Gamma\left(A+1-a\right)}{\Gamma\left(A+1\right)}a\Gamma\left(a\right)\frac{\theta}{a}\frac{\Gamma\left(A+1\right)}{\Gamma\left(A+1-a\right)}\frac{\Gamma\left(A+\theta-a\right)}{\Gamma\left(A+\theta\right)} = \\
&= \binom{J}{n}\sum_{A=1}^{J}\sum_{a=1}^{n}\overline{s}\left(n,a\right)\overline{s}\left(J-n,A-a\right)\frac{I^{A}}{\left(I\right)_J}\frac{1}{\binom{A}{a}}E\left[S_a|\theta,A\right] = \\
&= \sum_{j=1}^{J_{\mathrm{M}}}\sum_{A=1}^{J}\sum_{a=1}^{n}\frac{\binom{J}{n}}{\binom{A}{a}}\overline{s}\left(n,a\right)\overline{s}\left(J-n,A-a\right)\frac{I^{A}}{\left(I\right)_J}P_{\text{hyp}}[a|j,J_{\mathrm{M}},A]E\left[S_j|\theta,J_{\mathrm{M}}\right] = \\
&= \sum_{j=1}^{J_{\mathrm{M}}}P_{\text{hyp}}^{\text{DL}}\left[n|m,j,J_{\mathrm{M}},J\right]E\left[S_j|\theta,J_{\mathrm{M}}\right] \qquad \text{(C-1)}
\end{aligned}$$

where in the first line we have used the polynomial form of the Pochhammer symbol (3) and the definition of the beta function (4).

Appendix D. Derivation of (36) and (37)

First we note that we can write (35) as

$$P[\overrightarrow{D}|\Theta, J] = \prod_{i=1}^{S} \frac{P\left[n_i, ..., n_S|\theta, J_i\right]}{P\left[n_{i+1}, ..., n_S|\theta, J_{i+1}\right]} \tag{D-1}$$

No dispersal limitation

Without dispersal limitation, each term in (D-1) can be written as

$$\begin{aligned} \frac{P\left[n_i, ..., n_S|\theta, J_i\right]}{P\left[n_{i+1}, ..., n_S|\theta, J_{i+1}\right]} &= \frac{\frac{J_i!}{\prod_{k=i}^{S} n_k \prod_{j=1}^{J_i} \Phi_j!} \frac{\theta^{S-(i-1)}}{(\theta)_{J_i}}}{\frac{J_{i+1}!}{\prod_{k=i+1}^{S} n_k \prod_{j=1}^{J_i} (\Phi_j - \delta_{jn})!} \frac{\theta^{S-i}}{(\theta)_{J_{i+1}}}} = \\ &= \frac{1}{\Phi_{n_i}} \frac{\theta}{n_i} \frac{J_i!}{(J_i - n_i)!} \frac{(\theta)_{J_i - n_i}}{(\theta)_{J_i}} = \frac{E\left[S_{n_i}|\theta, J_i\right]}{\Phi_{n_i}} \end{aligned} \tag{D-2}$$

Here, δ_{jn} is Kronecker's delta which is equal to 1 if $j = n$ and 0 otherwise. Substituting this in (D-1) leads to (36). One can easily check that substituting (1) in (36) leads back to the multivariate probability distribution (2).

Dispersal limitation

When dispersal is limited, each term in (D-1) is given by

$$\frac{P\left[n_i, ..., n_S|\theta, m, J_i\right]}{P\left[n_{i+1}, ..., n_S|\theta, m, J_{i+1}\right]} = \frac{\frac{J_i!}{\prod_{k=1}^{S_i} n_k \prod_{j=1}^{J_i} \Phi_j!} \frac{\theta^{S-(i-1)}}{(I)_{J_i}} \sum_{A=S-(i-1)}^{J_i} K(\overrightarrow{D}_i, A) \frac{I^A}{(\theta)_A}}{\frac{J_{i+1}!}{\prod_{k=1}^{S_{i+1}} n_k \prod_{j=1}^{J_{i+1}} (\Phi_j - \delta_{jn})!} \frac{\theta^{S-i}}{(I)_{J_{i+1}}} \sum_{A=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, A) \frac{I^A}{(\theta)_A}} \tag{D-3}$$

The key step is now to realize that $K(\overrightarrow{D}_i, A)$ is a combination of terms involving $\frac{\overline{s}(n_i,j)\overline{s}(j,1)}{\overline{s}(n_i,1)}$ and $K(\overrightarrow{D}_{i+1}, k)$ where $k = A - j$ (Etienne 2005). Equation (D-3) then simplifies to, also cancelling equal terms in numerator and denominator,

$$P\left[n_i|\theta, m, J_i\right] = \frac{1}{\Phi_{n_i}} \frac{\theta}{n_i} \frac{J_i!}{(J_i - n_i)!} \frac{(I)_{J_i - n_i}}{(I)_{J_i}} \frac{\sum_{j=1}^{n_i} \frac{\overline{s}(n_i,j)\overline{s}(j,1)}{\overline{s}(n_i,1)} I^j \sum_{k=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, k) \frac{I^k}{(\theta)_{j+k}}}{\sum_{A=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, A) \frac{I^A}{(\theta)_A}} \tag{D-4}$$

We can now use the identities of the Stirling numbers of the first kind and the definition of the Beta function (4) to find

$$\begin{aligned} P\left[n_i|\theta, m, J_i\right] &= \frac{1}{\Phi_{n_i}} \theta \frac{J_i!}{n_i!\,(J_i - n_i)!} \frac{(I)_{J_i - n_i}}{(I)_{J_i}} \frac{\sum_{j=1}^{n_i} \overline{s}\left(n_i, j\right) I^j \sum_{k=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, k) \frac{I^k}{(\theta)_k} \frac{\Gamma(j)\Gamma(\theta+k)}{\Gamma(\theta+j+k)}}{\sum_{A=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, A) \frac{I^A}{(\theta)_A}} = \\ &= \frac{1}{\Phi_{n_i}} \theta \binom{J_i}{n_i} \frac{(I)_{J_i - n_i}}{(I)_{J_i}} \frac{\sum_{j=1}^{n_i} \overline{s}\left(n_i, j\right) I^j \sum_{k=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, k) \frac{I^k}{(\theta)_k} \int_0^1 x^{j-1} \left(1 - x\right)^{k+\theta-1} dx}{\sum_{A=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, A) \frac{I^A}{(\theta)_A}} = \\ &= \frac{1}{\Phi_{n_i}} \frac{\theta}{(I)_{J_i}} \binom{J_i}{n_i} (I)_{J_{i+1}} \frac{\int_0^1 \sum_{j=1}^{n_i} \overline{s}\left(n_i, j\right) (Ix)^j \sum_{k=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, k) \frac{(I(1-x))^k}{(\theta)_k} \frac{(1-x)^{\theta-1}}{x} dx}{\sum_{A=S-i}^{J_{i+1}} K(\overrightarrow{D}_{i+1}, A) \frac{I^A}{(\theta)_A}} \end{aligned} \tag{D-5}$$

Using (3), multiplying the integrand by $\frac{(I(1-x))_{J_{i+1}}}{(I(1-x))_{J_{i+1}}}$ and rearranging terms then leads to

$$P\left[n_i|\theta,m,J_i\right]=\frac{1}{\Phi_{n_i}}\frac{\theta}{(I)_{J_i}}\binom{J_i}{n_i}\int_0^1(Ix)_{n_i}\left(I\left(1-x\right)\right)_{J_i-n_i}\frac{(1-x)^{\theta-1}}{x}\frac{\sum_{k=S-i}^{J_{i+1}}\frac{K(\overrightarrow{D}_{i+1},k)}{(\theta)_k}\frac{(I(1-x))^k}{(I(1-x))_{J_{i+1}}}}{\sum_{A=S-i}^{J_{i+1}}\frac{K(\overrightarrow{D}_{i+1},A)}{(\theta)_A}\frac{I^A}{(I)_{J_{i+1}}}}dx \tag{D-6}$$

Defining

$$F\left(x|\theta,m,\overrightarrow{D}_{i+1}\right):=\frac{\sum_{k=S-i}^{J_{i+1}}\frac{K(\overrightarrow{D}_{i+1},k)}{(\theta)_k}\frac{(I(1-x))^k}{(I(1-x))_{J_{i+1}}}}{\sum_{A=S-i}^{J_{i+1}}\frac{K(\overrightarrow{D}_{i+1},A)}{(\theta)_A}\frac{I^A}{(I)_{J_{i+1}}}} \tag{D-7}$$

and using (39) and (21), we obtain our end result (37).

Appendix E. Derivation of (1) and (6) from (2) and (11)

No dispersal limitation

Equation (1) can be derived from (2) as follows. From (41) we get

$$\begin{aligned} E\left[S_n|\theta,J\right] &= \sum_{\Phi_n=1}^{J}\sum_{\left\{\vec{D}|\Phi_n\right\}}\Phi_n\frac{J!}{\prod_{i=1}^{S}n_i\prod_{j=1}^{J}\Phi_j!}\frac{\theta^S}{(\theta)_J} = \\ &= \sum_{\Phi_n=1}^{J}\sum_{\left\{\vec{D}|\Phi_n\right\}}\frac{J!}{\prod_{i=1}^{S}n_i\prod_{j=1}^{J}\left(\Phi_j-\delta_{jn}\right)!}\frac{\theta^S}{(\theta)_J} \end{aligned} \tag{E-1}$$

Defining $\Phi'_j := \Phi_j - \delta_{jn}$ where δ_{jn} is Kronecker's delta (which is equal to 1 if $j = n$ and 0 otherwise), we can rewrite this as a sum of probabilities of observing exactly $\Phi_n - 1$ species with abundance n in a subsample of size $J - n$,

$$\begin{aligned} E\left[S_n|\theta,J\right] &= \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{(\theta)_{J-n}}{(\theta)_J}\sum_{\Phi'_n=0}^{J-n}\sum_{\left\{\vec{D}|\Phi'_n\right\}}\frac{(J-n)!}{\prod_{i=1}^{S-1}n_i\prod_{j=1}^{J-n}\Phi'_j!}\frac{\theta^{S-1}}{(\theta)_{J-n}} = \\ & \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{(\theta)_{J-n}}{(\theta)_J}\sum_{\left\{\vec{D}\right\}}\frac{(J-n)!}{\prod_{i=1}^{S-1}n_i\prod_{j=1}^{J-n}\Phi'_j!}\frac{\theta^{S-1}}{(\theta)_{J-n}} \end{aligned} \tag{E-2}$$

The sum on the right hand side is the sum of the probabilities of over all possible datasets with sample size $J - n$, which, evidently, equals unity, and after expressing Pochhammer symbols as quotients of gamma functions, we obtain (1).

Also, the expected number of species (of any abundance) can be calculated as follows:

$$\begin{aligned} E\left[S|\theta,J\right] &= \sum_{S=1}^{J}SP\left[S|\theta,J\right] = \sum_{S=1}^{J}S\overline{s}\left(J,S\right)\frac{\theta^S}{(\theta)_J} = \\ &= \theta\sum_{S=1}^{J}\overline{s}\left(J,S\right)\frac{S\theta^{S-1}}{(\theta)_J} = \theta\sum_{S=1}^{J}\overline{s}\left(J,S\right)\frac{1}{(\theta)_J}\frac{d}{d\theta}\theta^S = \\ &= \theta\frac{d}{d\theta}\left(\sum_{S=1}^{J}\overline{s}\left(J,S\right)\frac{\theta^S}{(\theta)_J}\right) - \theta\sum_{S=1}^{J}\overline{s}\left(J,S\right)\theta^S\frac{d}{d\theta}\frac{1}{(\theta)_J} = -(\theta)_J\frac{d}{d\theta}\frac{1}{(\theta)_J} = \\ &= \theta\left(\Psi\left(\theta+J\right)-\Psi\left(\theta\right)\right) = \sum_{i=1}^{J}\frac{\theta}{\theta+i-1} \end{aligned} \tag{E-3}$$

where $\Psi(x)$ is the digamma function or psi function, $\Psi(x) = \frac{d}{dx}\ln\Gamma(x) = \frac{1}{\Gamma(x)}\frac{d}{dx}\Gamma(x)$ and we have used the identity $\sum_{S=1}^{J}\overline{s}\left(J,S\right)\frac{\theta^S}{(\theta)_J} = 1$.

Dispersal limitation

When dispersal is limited, with (41) we can derive (6) from (11):

$$\begin{aligned} E\left[S_n|\theta,m,J\right] &= \sum_{\Phi_n=0}^{S} \Phi_n \sum_{\left\{\overrightarrow{D}|\Phi_n\right\}} P[\overrightarrow{D}|\theta,m,J] = \\ &= \sum_{\Phi_n=1}^{S}\sum_{\left\{\overrightarrow{D}|\Phi_n\right\}} \Phi_n \frac{J!}{\prod_{i=1}^{S} n_i \prod_{j=1}^{J} \Phi_j!}\frac{\theta^S}{(I)_J}\sum_{A=S}^{J} K(\overrightarrow{D},A)\frac{I^A}{(\theta)_A} = \\ &= \sum_{\Phi_n=1}^{S}\sum_{\left\{\overrightarrow{D}|\Phi_n\right\}} \frac{J!}{\prod_{i=1}^{S} n_i \prod_{j=1}^{J} (\Phi_j-\delta_{jn})!}\frac{\theta^S}{(I)_J}\sum_{A=S}^{J} K(\overrightarrow{D},A)\frac{I^A}{(\theta)_A} \end{aligned} \tag{E-4}$$

As in the case without dispersal limitation, we define $\Phi_j' := \Phi_j - \delta_{jn}$ and we work towards a sum of probabilities of observing exactly $\Phi_n - 1$ species with abundance n in a sample of size $J - n$ (the corresponding dataset is called $\overrightarrow{D}'$),

$$E\left[S_n|\theta,m,J\right] = \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\sum_{\Phi_n'=0}^{S-1}\sum_{\left\{\overrightarrow{D}|\Phi_n'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{A=S}^{J} K(\overrightarrow{D},A)\frac{I^A}{(\theta)_A} =$$

$$\begin{aligned} &= \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\sum_{\Phi_n'=0}^{S-1}\sum_{\left\{\overrightarrow{D}'|\Phi_n'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{j=1}^{n}\frac{\overline{s}\,(n,j)\,\overline{s}\,(j,1)}{\overline{s}\,(n,1)}I^j\sum_{k=S-1}^{J-n} K(\overrightarrow{D}',k)\frac{I^k}{(\theta)_{j+k}} = \\ &= \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\sum_{\left\{\overrightarrow{D}'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{j=1}^{n}\frac{\overline{s}\,(n,j)\,\overline{s}\,(j,1)}{\overline{s}\,(n,1)}I^j\sum_{k=S-1}^{J-n} K(\overrightarrow{D}',k)\frac{I^k}{(\theta)_k}\frac{(\theta)_k}{(\theta)_{j+k}} = \\ &= \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\sum_{\left\{\overrightarrow{D}'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{j=1}^{n}\frac{\overline{s}\,(n,j)}{\overline{s}\,(n,1)}I^j\sum_{k=S-1}^{J-n} K(\overrightarrow{D}',k)\frac{I^k}{(\theta)_k}\frac{\Gamma(j)\,\Gamma(\theta+k)}{\Gamma(\theta+j+k)} \end{aligned} \tag{E-5}$$

The term with Gamma functions can be written as a Beta function, and the Beta function can be written in its integral notation:

$$\begin{aligned} E\left[S_n|\theta,m,J\right] &= \frac{\theta}{n}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\times \\ &\times \sum_{\left\{\overrightarrow{D}'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{j=1}^{n}\frac{\overline{s}\,(n,j)}{\overline{s}\,(n,1)}I^j\sum_{k=S-1}^{J-n} K(\overrightarrow{D}',k)\frac{I^k}{(\theta)_k}\int_0^1 x^{j-1}\,(1-x)^{k+\theta-1}\,dx \end{aligned} \tag{E-6}$$

Changing the order of integration and summation, and using (3), we obtain

$$
\begin{aligned}
&E\left[S_n|\theta,m,J\right] = \frac{\theta}{n!}\frac{J!}{(J-n)!}\frac{1}{(I)_J}\times \\
&\times\int_0^1 \sum_{\left\{\overrightarrow{D}'\right\}} \frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\theta^{S-1}\sum_{j=1}^{n}\overline{s}\left(n,j\right)\left(Ix\right)^j \sum_{k=S-1}^{J-n} K(\overrightarrow{D}',k)\frac{\left(I\left(1-x\right)\right)^k}{\left(\theta\right)_k}\frac{\left(1-x\right)^{\theta-1}}{x}dx \\
&=\theta\binom{J}{n}\frac{1}{(I)_J}\times \\
&\times\int_0^1 \left(Ix\right)_n\left(I\left(1-x\right)\right)_{J-n}\frac{\left(1-x\right)^{\theta-1}}{x}\sum_{\left\{\overrightarrow{D}'\right\}}\frac{(J-n)!}{\prod_{i=1}^{S-1} n_i \prod_{j=1}^{J}\Phi_j'!}\frac{\theta^{S-1}}{\left(I\left(1-x\right)\right)_{J-n}}\sum_{k=S-1}^{J-n}K(\overrightarrow{D}',k)\frac{\left(I\left(1-x\right)\right)^k}{\left(\theta\right)_k}dx \\
&=\theta\binom{J}{n}\frac{1}{(I)_J}\int_0^1\left(Ix\right)_n\left(I\left(1-x\right)\right)_{J-n}\frac{\left(1-x\right)^{\theta-1}}{x}dx
\end{aligned}
\tag{E-7}
$$

where in the last line we have used the fact that the sum of the probabilities of all possible datasets equals unity.

Also, the expected number of species (of any abundance) can be calculated as follows:

$$
\begin{aligned}
E\left[S|\theta,m,J\right] &= \sum_{S=1}^{J} SP\left[S|\theta,mJ\right] = \sum_{S=1}^{J} S\sum_{A=S}^{J}\overline{s}(J,A)\frac{I^A}{(I)_J}\overline{s}(A,S)\frac{\theta^S}{(\theta)_A} = \\
&= \sum_{A=1}^{J}\overline{s}(J,A)\frac{I^A}{(I)_J}\sum_{S=1}^{A}S\overline{s}(A,S)\frac{\theta^S}{(\theta)_A} = \\
&= \sum_{A=1}^{J}\overline{s}(J,A)\frac{I^A}{(I)_J}\sum_{i=1}^{A}\frac{\theta}{\theta+i-1} = \\
&= \sum_{i=1}^{J}\frac{\theta}{\theta+i-1}\sum_{A=i}^{J}\overline{s}(J,A)\frac{I^A}{(I)_J} = \\
&= \sum_{i=0}^{J-1}\frac{\theta}{\left(\theta+i\right)(I)_J}\sum_{A=i+1}^{J}\overline{s}(J,A)I^A
\end{aligned}
\tag{E-8}
$$

Appendix F. A historical note on the origins of the binomial and hypergeometric distributions

The first occurrence of “binomial distribution” is found in Yule (1911) on p. 305: “The binomial distribution only becomes approximately normal when n is large, and this limitation must be remembered in applying the table to cases in which the distribution is strictly binomial”. Fisher (1925) adopted the distribution (part III, section 18). Although, the name for the distribution relatively new, the distribution itself has been studied since Bernoulli (1713, part 1.)

The earliest use of “hypergeometric distribution” appears in the title of Gonin (1936). Again, the name is relatively recent but the distribution itself is already suggested in Problem IV of Huygens (1657, p. 12). He and several other mathematicians solved the problem, whereas Bernoulli and de Moivre gave solutions for the general case (Hald 2003). At the end of the 19th. century Pearson (1899) wrote a paper in which he considered fitting the distribution (given by the ”hypergeometrical series”) to data.

This brief historical note is based on http://members.aol.com/jeff570/mathword.html.

References

Bernoulli, J. (1713). *Ars Conjectandi*. Basel, Switzerland. English translation available at http://cerebro.xu.edu/math/Sources/JakobBernoulli/ars_sung/ars_sung.html.

Fisher, R.A. (1925). *Statistical methods for research workers*. London, U.K.: Oliver & Boyd. Available at http://psychclassics.yorku.ca/Fisher/Methods/index.htm

Gonin, H.T. (1936). The use of factorial moments in the treatment of the hypergeometric distribution and in tests for regression. *Philosophical Magazine* 7: 215-226.

Hald (2003). A history of probability and statistics and their applications before 1750. New York, N.Y.: Wiley-Interscience.

Huygens, C. (1657). *De ratiociniis in ludo aleae*. Reprint of an English translation available at http://www.leidenuniv.nl/fsw/verduin/stathist/huygens/huyg1714p.pdf

Pearson, K. (1899). On certain properties of the hypergeometrical series, and on the fitting of such series to observation polygons in the theory of chance. *Philosophical Magazine* 47: 236-246.

Yule, G.U. (1911). *An introduction to the theory of statistics*. London, U.K.: Charles Griffin & Co. Ltd.